Potential Scattering as Opposed to Scattering Associated with Independent Particles in the S-Matrix Theory of Strong Interactions*

GEOFFREY F. CHEW AND STEVEN C. FRAUTSCHI
Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California
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A definition of a relativistic generalized potential is given, suitable at arbitrary energies for a pair of particles whose elastic scattering amplitude satisfies the Mandelstam representation. It is shown that the generalized potential plays a role in the dynamics analogous to that of the ordinary nonrelativistic potential in a Schrödinger equation and determines the scattering to the same extent. Below the threshold for inelastic processes the generalized potential is real and its energy dependence in the elastic region is expected for certain particle combinations (such as the nucleon-nucleon) to be weak. In such cases one may uniquely define, for use in the Schrödinger equation, an energy-independent ordinary potential that coincides with the potential of Charap and Fubini. In general, when the potential is complex and energy-dependent the dynamical problem involves iteration of an integral equation deduced by Mandelstam. The generalized potential may be decomposed according to range and it is shown that keeping only the long- and medium-range parts, corresponding to transfer of one or two particles, is almost equivalent to the "strip approximation." Finally, a general definition is given of "pure potential scattering" as opposed to scattering associated with "independent" particles, either stable or unstable, and a variety of experimental situations are discussed with respect to this distinction, which is shown to be susceptible to experimental test.

WE should like to propose in terms of the analytic continuation of the S matrix a relativistic definition of a generalized two-body "potential" that appears to have three useful properties: (a) Its role in the dynamics is analogous to that of an ordinary potential, and in the nonrelativistic limit its relation to the potential defined by Charap and Fubini[1] may be established. (b) Its long-range and medium-range parts may be evaluated for arbitrary energies in terms of one- and two-body S-matrix elements. (c) It allows a precise and physically helpful distinction between pure "potential scattering" and scattering associated with "independent" particles.

Our definition is made within the Mandelstam framework, which describes the scattering amplitude for three different two-body reactions by a single analytic function.[2] Suppose the two particles whose mutual interaction is of interest are called a and b. Then we label the three Mandelstam channels as follows:

I. $a+b\rightarrow a+b$ (barycentric energy squared$=s$),

II. $a+\bar{a}\rightarrow b+\bar{b}$ (barycentric energy squared$=t$),

III. $a+\bar{b}\rightarrow a+\bar{b}$ (barycentric energy squared$=u$).

Roughly speaking, channel II provides the "direct" forces for channel I, while channel III provides "exchange" forces. More precisely, we define the generalized direct potential $V_{\mathrm{I}}^{\mathrm{II}}(t,s)$ as the channel II absorptive part, A_2, *minus* the contribution from $\rho_{st}^{\mathrm{I(el)}}$, the elastic double-spectral function for channel I. That is,

$$V_{\mathrm{I}}^{\mathrm{II}}(t,s)=A_2(t,s)-(1/\pi)\int\int ds'[\rho_{st}^{\mathrm{I(el)}}(s',t)/s'-s]. \quad (1)$$

The generalized exchange potential has a corresponding definition in terms of channel III and $\rho_{su}^{\mathrm{I(el)}}$.

The elastic double-spectral function for channel I is given by the Cutkosky graphs of Fig. 1.[3] If we diagonalize the S matrix in channel I with respect to all internal quantum numbers (isotopic spin, strangeness, etc.), then the Cutosky recipe[3] gives us (if spin complications are ignored)

$$\rho_{st}^{\mathrm{Iel}}(s,t)=\frac{1}{\pi}\frac{1}{q_s\sqrt{s}}\int\int dt'dt''\frac{A_2^*(t',s)A_2(t'',s)}{K^{\frac{1}{2}}(q_s^2;t,t',t'')}+\frac{1}{\pi}\frac{1}{q_s\sqrt{s}}\int\int du'du''\frac{A_3^*(u',s)A_3(u'',s)}{K^{\frac{1}{2}}(q_s^2;t,u',u'')}, \quad (2)$$

where

$$K(q^2,t,t',t'')=t^2+t'^2+t''^2-2(tt'+tt''+t't'')-tt't''/q^2,$$

with a corresponding formula for $\rho_{su}^{\mathrm{I(el)}}(s,u)$ in which t is replaced by u and the bilinear combinations $A_2^*A_3$ and $A_3^*A_2$ appear. The range of integration in (2) is restricted to the region in which K is positive. These formulas were first derived by Mandelstam from the elastic unitarity condition.[2]

Now, by definition,

$$A_2(t,s)=V_{\mathrm{I}}^{\mathrm{II}}(t,s)+(1/\pi)\int ds'[\rho_{st}^{\mathrm{I(el)}}(s',t)/s'-s],$$

$$A_3(u,s)=V_{\mathrm{I}}^{\mathrm{III}}(u,s)+(1/\pi)\int ds'[\rho_{su}^{\mathrm{I(el)}}(s',u)/s'-s], \quad (3)$$

so if the generalized potentials $V_{\mathrm{I}}^{\mathrm{II}}$ and $V_{\mathrm{I}}^{\mathrm{III}}$ are given one may compute the elastic double-spectral functions by iteration of formula (2), as originally emphasized by Mandelstam.[2] All statements to this point have been

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] J. Charap and S. Fubini, Nuovo cimento **14**, 540 (1959).

[2] S. Mandelstam, Phys. Rev. **112**, 1344 (1958).

[3] R. E. Cutkosky, J. Math. Phys. **1**, 429 (1960); Phys. Rev. Letters **4**, 624 (1960).

formal and, correspondingly, exact. We now consider, in order, the three aspects of our generalized potential that were listed in the first paragraph.

(a) If one considers nonrelativistic scattering by a superposition of Yukawa direct and exchange potentials[4]:

$$V_{\rm dir}(r)=-\int dt\, g_{\rm dir}(t)\frac{e^{-r\sqrt{t}}}{r},$$
$$V_{\rm ex}(r)=-\int du\, g_{\rm ex}(u)\frac{e^{-r\sqrt{u}}}{r}, \tag{4}$$

then, except for trivial questions of normalization, if $V_{\rm I}^{\rm II}(t,s)$ is replaced by $g_{\rm dir}(t)$ and $V_{\rm I}^{\rm III}(u,s)$ by $g_{\rm ex}(u)$, the equations determining the double-spectral functions differ from Eqs. (2) and (3) above only in the replacement of the factor $\sqrt{s}$ by M_a+M_b. Thus our generalized potentials determine the dynamics to the same extent and in much the same way as an ordinary potential. In fact it is easy to show that although the "potential" defined by Eq. (1) is in general energy-dependent and complex, it becomes real for s below the inelastic threshold, and when M_a^2, $M_b^2\gg m_\pi^2$, as in nucleon-nucleon scattering, the dependence on s in the elastic region is weak (especially for small t). It has been pointed out to us by J. Charap that in such a case, if one wishes to use the potential in a Schrödinger equation, some modification is required because even when q_s^2 is small there are contributions from large values of s' in the integral on the right-hand side of Eq. (3). Such contributions behave like an extra term in a nonrelativistic potential, and can be calculated in a simple way from a knowledge of the generalized potential.[5] Once this is done, one obtains the nonrelativistic potential already defined by Charap and Fubini.[1] Of course if one works directly with the integral equations (2) and (3), as one must in π-π and π-N scattering, this modification is unnecessary.

(b) The Cutkosky recipe[3] tells us how to compute

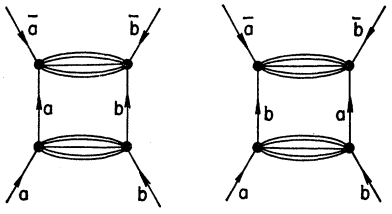

FIG. 1. Cutkosky graphs for $\rho_{st}^{\rm I(el)}$. To obtain $\rho_{su}^{\rm I(el)}$ the lines for $\bar{a}$ and $\bar{b}$ should be interchanged.

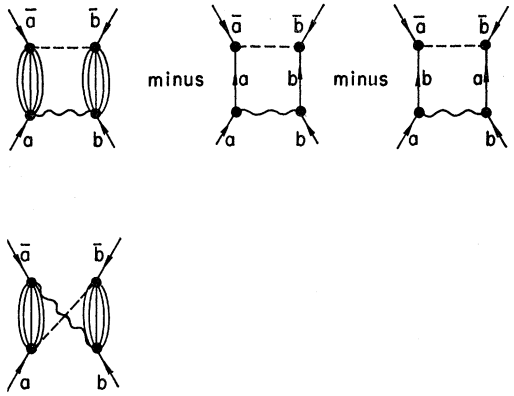

FIG. 2. Cutkosky graphs providing double-spectral functions for the two-body "direct potential." A subtraction term corresponding to $J=0$ in channel II must also be included.

the generalized potentials in terms of analytic continuations of S-matrix elements. It is convenient to classify contributions according to "range," that is, in terms of the masses of the various intermediate states in channels II and III. The very-long-range one-particle contributions are trivial, as usual, and require no special comment. The medium-range two-particle contributions to $V_{\rm I}^{\rm II}$ are associated with the graphs of Fig. 2, where the omission of the box diagrams should be noted. The latter are contained in $\rho_{st}^{\rm I(el)}$ and must be eliminated according to formula (1). Formulas for the graphs of Fig. 2 will be similar to formula (2) above, with s and t interchanged and a K function that depends on the masses of the exchanged particles. Thus, a sufficient knowledge of the absorptive parts for the appropriate two-body reactions will allow a calculation of the part of the direct potential associated with two-particle transfer; the exchange potential can be handled in a similar way. Keeping only the one- and two-particle contributions to the generalized potential is essentially equivalent to the "strip approximation" described in an earlier Letter.[6] The equivalence is not precise because, for example, in the strip approximation one includes terms in the direct potential of the type of Fig. 3, which correspond to the transfer of more than two particles. If we have made a physically correct identification of the range concept in defining our generalized potentials it appears superfluous to carry such terms when multi-particle transfer is not systematically calculated. (On the other hand, their inclusion should do no harm and may give some estimate of the importance of short-range forces.)

We may comment here that the approximation of neglecting or treating phenomenologically the exchange of more than two particles does not depend for its validity on the energy. The approximation appears just as plausible at high as at low energies, so, as explained previously,[6] the "peripheral" approach is by no means

[4] R. Blankenbecler, M. L. Goldberger, N. N. Khuri, and S. B. Treiman, Ann. Phys. **10**, 62 (1960).

[5] The correction may be calculated up to any finite value of t by a finite number of iterations of Eq. (3), taking the difference between relativistic and nonrelativistic values of the integral as the correction to the potential. In practice, in the N-N problem, if one wishes to calculate only the one- and two-pion parts of the potential ($t<9m_\pi^2$) a single iteration suffices. We are extremely grateful to M. Froissart for assistance in clarifying this question.

[6] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **5**, 580 (1960); Phys. Rev. **123**, 1478 (1961).

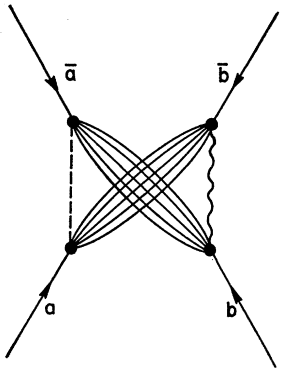

FIG. 3. Cutkosky graph included in the strip approximation but corresponding to the transfer of more than two particles in the generalized direct potential.

restricted to the elastic region. Our direct "potential," for example, automatically develops an imaginary part equal to $\rho_{st}^{\mathrm{I(in)}}$ for s above the inelastic threshold, where

$$\rho_{st}^{\mathrm{I(in)}}=\rho_{st}-\rho_{st}^{\mathrm{I(el)}}.$$

This imaginary part causes the proper reduction in the modulus of the scattered wave to compensate for inelastic processes. However, the mechanism is not quite the same as for a complex energy-independent potential in a Schrödinger equation, where time-reversal is sacrificed. Energy dependence is intimately associated with the imaginary part of our generalized potential, and time reversal is thereby preserved.

(c) We finally discuss a matter of principle, assuming that somehow $V_{\mathrm{I}}^{\mathrm{II}}$ and $V_{\mathrm{I}}^{\mathrm{III}}$ are completely known. Our comments here are a synthesis of remarks made at various times by others but never, to our knowledge, collected in one place. It has been shown above that a knowledge of the generalized potential for channel I is equivalent to a complete knowledge of the double-spectral functions as well as a knowledge of the single-spectral functions for channels II and III. In order to define the amplitude completely, however, we further require, in addition to any over-all subtraction constants, the single-spectral function for channel I (i.e., the elastic absorptive parts of a finite number of low partial waves in channel I). Here there arises the well-known CDD ambiguity,[7] which is now recognized as equivalent to the possibility of unstable elementary particles with the quantum numbers of channel I. One way to characterize the ambiguity is in terms of the N/D technique of partial-wave calculation introduced by Chew and Mandelstam.[8] Let us briefly review the essential features of this method.

One starts with a knowledge of the *complete* discontinuity across the unphysical cuts of the partial-wave amplitude and of the *inelastic* discontinuity on the physical cut. All this information can be obtained once our generalized potentials are given and the dynamical Eqs. (2) and (3) have been solved by iteration. The inelastic discontinuity is obtained directly by projection from the imaginary part of the generalized potentials,

$$\mathrm{Im}A_l^{\mathrm{I(in)}}(s)=\frac{1}{2}\int_{-1}^{+1}\mathrm{d}\cos\theta\ P_l(\cos\theta)\times\left[\frac{1}{\pi}\int dt'\frac{\mathrm{Im}V_{\mathrm{I}}^{\mathrm{II}}(t',s)}{t'-t}+\frac{1}{\pi}\int du'\frac{\mathrm{Im}V_{\mathrm{I}}^{\mathrm{III}}(u',s)}{u'-u}\right],$$

while the complete unphysical discontinuity of A_l^{I} is to be projected in the usual way out of A_2 and A_3.[8] Froissart has shown how an equivalent purely elastic problem can then be constructed, with a modified discontinuity on the unphysical cut and a modified phase shift.[9] We make our discussion here in terms of this modified problem, where the elastic unitarity condition is exact. The denominator function may then be *defined* by

$$D_l(s)=\exp\left(-\frac{s-s_0}{\pi}\int_{s_0}^{\infty}ds'\frac{\delta_l(s')}{(s'-s_0)(s'-s)}\right),\quad(5)$$

where δ_l is the equivalent elastic phase shift $[\delta_l(s_0)=0]$, and separate dispersion relations may be written down for numerator and denominator functions.[10] These dispersion relations, once established, lead to a linear integral equation, singular at most at infinity, whose solution if it exists at all is unique for the given discontinuities,[8] but there is an ambiguity with respect to the number of subtractions. Unitarity restricts the asymptotic behavior of the quotient N_l/D_l, but an arbitrary number n of subtractions in D_l is possible, provided it is matched by a corresponding number in N_l. It is possible to associate the $2n$ subtraction constants with the positions and residues of n pairs of poles on the unphysical sheet.[11] These we wish to call unstable "independent" particles. Poles on the unphysical sheet that occur even when no "extra" subtractions are made we wish to call "dynamical resonances."

For nonrelativistic scattering an unambiguous distinction can be made.[4] If one wants the N/D solution that corresponds to "pure" potential scattering one writes the dispersion relation for D_l with no arbitrary constants:

$$D_l(s)=1+\frac{s-s_0}{\pi}\int_{s_0}^{\infty}ds'\frac{\mathrm{Im}D_l(s')}{(s'-s_0)(s'-s)}.\quad(6)$$

We propose that this prescription be extended to the

[7] L. Castillejo, R. Dalitz, and F. Dyson, Phys. Rev. **101**, 453 (1956).

[8] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

[9] M. Froissart (to be published).

[10] A recent preprint by R. Omnes gives an alternative approach entirely in terms of the function $D_l(s)$. The content of the two approaches is equivalent, although the Omnes method has the advantage of exhibiting explicitly the unphysical sheet.

[11] The $2n$ constants have some restrictions on their range of values, corresponding to the requirement that the poles not appear on the physical sheet.

relativistic case as a *definition* of pure potential scattering. Such a solution (if it exists) is completely determined once the generalized potential is given, and corresponds to the condition $\delta_l \to \delta_l(\infty)$ as $s \to \infty$, where $\delta_l(\infty) < \pi$, as may be seen from Eq. (5):[12]

$$\lim_{s\to\infty} D_l(s) = \exp\left[\frac{\delta_l(\infty)}{\pi} \ln s + \text{constant}\right] \propto s^{\delta_l(\infty)/\pi}. \quad (7)$$

Evidently, making n arbitrary subtractions in D_l corresponds to

$$n\pi \leq \delta_l(\infty) < (n+1)\pi.$$

To complete the argument, we consider the possibility of a stable particle with the quantum numbers of channel I. With a sufficiently attractive force, a zero of the function $D_l(s)$ given by Eq. (6) may move to the real axis of the physical sheet at $s = s_b, s_b < s_0$, corresponding to a bound state of mass $\sqrt{s_b}$. [In such a case the function $D_l(s)$ given by Eq. (6) differs from that defined in Eq. (5) by a factor $(s-s_b)/(s_0-s_b)$.] The position and residue of the pole in the amplitude associated with this zero are completely determined by the generalized potential. In contrast, a stable "independent" particle of mass $\sqrt{s_p}$ brings with it two new constants. One may either introduce a pole into the numerator function at $s = s_p$ or make a single subtraction in both N_l and D_l and adjust the two constants to make $D_l(s_p) = 0$ while at the same time $N_l(s_p)$ yields the desired residue of the pole. These two ways of constructing the amplitude are easily shown to give exactly the same result. (The former yields the function $D_l(s)$ defined by Eq. (5) while the denominator function from the latter differs by a factor $(s-s_p)/(s_0-s_p)$; however the two numerator functions differ by exactly the same factor.) The former method is more convenient from a computational standpoint, but the latter shows clearly that a CDD pole is an independent particle that happens to be unstable.

We are aware that when unitarity in channels II and III is included in the discussion (as well as crossing symmetry, if one or both of these channels correspond to the same reaction as channel I), then the inclusion or exclusion of unstable independent particles in channel I is not completely arbitrary. For example, Froissart has recently shown that over-all unitarity (in all three channels) uniquely determines all but the S and P waves once the complete double-spectral function is given.[13] Even the P wave is determined if one accepts the Pomeranchuk high-energy relations, and if the notion of "saturation" of the unitarity condition[6] is added there may be no arbitrariness left in the S matrix. Nevertheless it seems to us helpful to have a clean definition of pure potential scattering that can be applied in all situations. The prescription proposed here has often been privately discussed by workers in the field,[14] but never with relation to a precise definition of the "potential."

To illustrate the significance of our criterion let us consider some specific pairs of strongly interacting particles. Historically, the first interaction to be studied was that between two nucleons, and it has been found that an energy-independent potential, employed in a Schrödinger equation, gives an approximately correct description. Thus we expect that when the S-matrix approach is employed there will be no need for "independent" particles of nucleon-number two. The deuteron, of course, is a dynamical bound state. The pion-nucleon interaction has also been studied in detail and in the state of $I=\frac{3}{2}$, $J=\frac{3}{2}$ there is a resonance. The Chew-Low formula for the $(\frac{3}{2},\frac{3}{2})$ phase shift corresponds to the prescription of Eq. (6),[15] so its success in predicting the width of the resonance implies that we are not dealing here with an independent particle. The long-range attractive exchange-force due to single-nucleon transfer is chiefly responsible. By contrast, in the πN state with $I=\frac{1}{2}$, $J=\frac{1}{2}$ there may be a stable independent-particle pole in the sense of our definition, associated with the nucleon. This is not known to be a dynamical bound state, but the necessity for such a pole could be inferred from crossing symmetry once the generalized potential, with its single-nucleon exchange term, is given. The position and residue are therefore determined by the generalized potential.

Passing on to the $\pi\pi$ interaction, there have been two resonances indicated experimentally, one at an energy $\sim 2m_\pi$ with $I=0$, $J=0$ and one at $\sim 5m_\pi$ with $I=1$, $J=1$.[16] Some theories treat either or both of these unstable particles as independent of the pion, but the calculations of Chew and Mandelstam[8] and their proposed extension by the present authors,[6] for example, have been based on Eq. (6) and therefore correspond to the assumption that such states are dynamical resonances of the 2π system. It remains to be seen whether such calculations will be successful in predicting the positions and widths of the resonances or whether these parameters must be introduced as independent constants. A similar remark may be made about the resonance in πK scattering.[16]

In systems such as the $\pi\Lambda$, where the first inelastic process is also a two-body reaction (KN), caution must be exercised in applying the above simple criterion. The most satisfactory approach is to employ a matrix N/D method[17] including all the competing two-body reactions and treating only multi-particle states as inelastic. In such a spirit almost all models for the Y^*

[12] This asymptotic behavior was pointed out to us in a private conversation by M. Froissart (Physics Department, University of California, Berkeley).

[13] M. Froissart, Phys. Rev. **123**, 1053 (1961).

[14] For example, by M. Gell-Mann and S. Mandelstam.

[15] G. F. Chew and F. Low, Phys. Rev. **101**, 1570 (1956).

[16] For a survey of recently discovered resonances, see the Proceedings of the 1960 Berkeley Conference on Strong Interactions [Revs. Modern Phys. **33**, July (1961)].

[17] J. D. Bjorken, Phys. Rev. Letters **4**, 473 (1960).

currently under discussion would be classed as dynamical. If, however, one concentrates on the $\pi\Lambda$ system then the Dalitz-Tuan model makes the Y^* an independent particle, whereas the model based on global symmetry rests on an attractive force between the π and the Λ.[16]

The chief reason for writing this paper is our impression, perhaps erroneous, that many workers in the strong-interaction field do not realize that in the S-matrix framework any distinction can be made between different types of resonance elastic scattering. The distinction emphasized here, even if the words used in the description do not appear appropriate to all readers, is subject to experimental test. A closing observation that we find difficult to resist is that to date most such tests point *away* from the notion of independent-particle scattering. It is plausible, therefore, that *none* of the strongly interacting particles are *completely* independent but that each is a dynamical consequence of interactions between others. In such a situation, when the entire S matrix is considered, there would be no arbitrary dimensionless (coupling) constants and presumably only one dimensional constant to establish the scale of masses. However, when one concentrates on a small part of the over-all problem, asking about the elastic interaction of a particular pair of particles, the discussion presented here should still be meaningful.

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Radiative Decay of the Neutral K Meson: $K^0 \rightarrow \gamma+\gamma$†

J. DREITLEIN* AND H. PRIMAKOFF
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania

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The consequences of the particle mixture theory of the neutral K meson are investigated for the rare radiative decay mode: $K^0 \rightarrow \gamma+\gamma$. The two photon decay rates of the K_1^0, K_2^0 mesons are estimated as $\approx 1.3\times 10^5$ sec^{-1} (Cabibbo and Ferrari) and $\approx\{1.6\times 10^5/(g_{\Sigma nK}^2/4\pi)\}$ sec^{-1} $\approx 10^5$ sec^{-1}. It is shown that a time-dependent net circular polarization of each of the two photons results from the interference between the K_1^0 and K_2^0 channels feeding the 2γ state. The correlated linear polarizations of the two photons also exhibit a similar time-dependent behavior. The possibility of experimental detection of the effects discussed, from which the sign as well as the magnitude of the K_1^0, K_2^0 mass difference can be determined, is very briefly explored.

I

SOME unusual properties of the neutral K meson complex were first predicted by Gell-Mann and Pais[1]: the double lifetime behavior, and by Pais and Piccioni[2]: the regeneration phenomenon. Such properties have since been observed experimentally,[3,4] and theoretically have been shown to hold independently of the failure of parity conservation and of charge conjugation conservation in decays induced by the weak interactions.[5,6]

One further phenomenon peculiar to the neutral K mesons is the predicted time dependence of the rate of appearance of the neutral K-derived leptons,[7] an oscillatory effect occurring with a frequency given by the mass difference $\Delta m \equiv m_1 - m_2$ between the K_1^0 and K_2^0. Analogous oscillatory effects associated with the regeneration phenomenon and the double lifetime behavior have been used[4] to fix the order of magnitude of $|\Delta m|$ as $|\Delta m| \approx 1/\tau(K_1^0)$.

In this note we point out another curious neutral K phenomenon which is encountered in the rare radiative decay mode: $K^0 \rightarrow \gamma+\gamma$.

II

With respect to the strong interactions, the neutral K mesons are best described by one non-Hermitian field—that of the K^0 and $\bar{K}^0$ mesons—while, with respect to the weak interactions, two Hermitian fields—those of the K_1^0 and K_2^0 mesons—best characterize the neutral K meson decays. Immediately after a K^0 is created the corresponding state can be viewed as a coherent mixture of a K_1^0 and a K_2^0:

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* National Science Foundation Postdoctoral Fellow, 1960–1961

[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

[2] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955); see also M. L. Good, *ibid.* **106**, 591 (1957).

[3] K. Lande, E. Booth, J. Impeduglia, L. Lederman, and W. Chinowsky, Phys. Rev. **103**, 1901 (1956).

[4] F. Muller, R. W. Birge, W. B. Fowler, R. H. Good, W. Hirsch, R. P. Matsen, L. Oswald, W. M. Powell, H. White, and O. Piccioni, Phys. Rev. Letters **4**, 418 (1960) and *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960); R. W. Birge, R. P. Ely, W. M. Powell, H. Huzita, W. F. Fry, J. A. Gaides, V. Natali, R. B. Willman, and U. Camerini, *ibid.*

[5] R. Gatto, Phys. Rev. **106**, 168 (1957).

[6] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).

[7] S. B. Treiman and R. Sachs, Phys. Rev. **103**, 1545 (1956); see also S. B. Treiman and S. Weinberg, *ibid.* **116**, 239 (1959) for a discussion of analogous effects in the rate of appearance of neutral K derived 3π states.